An Improved Sliding-Load Calibration Procedure Using a Semiparametric Circle-Fitting Procedure

Gerd Vandersteen, Yves Rolain, *Member, IEEE,* Johan Schoukens, *Fellow, IEEE,* and Ann Verschueren

***Abstract*— Circle-fitting problems often occur in microwave engineering when dealing with variable delays, e.g., during calibration using a sliding load. This paper proposes an efficient semiparametric circle-fitting procedure, which takes into account the phase relationships over the frequencies. It produces more accurate results than the standard sliding-load calibration, requires only three positions on the sliding load for the whole frequency band, and is more robust to the settings of the positions of the sliding load. The proposed method also has the ability to detect whether or not the sliding load is defective or out of its specifications. This can be done by using only three positions on the sliding load. Optimal-position settings are then proposed. The performance of the proposed method is illustrated on sliding-load measurements up to 50 GHz, demonstrating the ability of detecting modeling errors and showing that the accuracy of the proposed method using three positions is comparable to the standard method with six positions.**

***Index Terms*— Calibration, estimation.**

I. Introduction

Microwave measurements often require the fitting of circles to the measurements of a sliding load to determine the reference characteristic impedance [1]. The present calibration method used in microwave-network analyzers fits a circle on the data without exploiting the frequency dependence of the transmission line. This solution normally works well for a sufficiently large number of positions on the sliding load. Every positioning of the sliding load requires a manual intervention and the number of positionings should, therefore, be reduced to decrease the cost. Furthermore, it is known that if only a small number of positions are available, imperfections of the sliding load are not easily detectable using this standard method. Since sliding loads are fragile and difficult to realize from mechanical and electromagnetical points of view, it is of high value to be able to detect possible defects in the calibration element.

This paper introduces a new calibration procedure, which takes the frequency dependence of the transmission line into account using a model for the delay. The main advantages of this new approach are as follows.

- The number of positions of the sliding load can be reduced to three without losing accuracy with respect to the standard method with six positions.
- Defects on the sliding load can be detected through a model validation technique, i.e., one can easily check whether or not the measurements satisfy the model of the sliding load.
- The maximum delay, and therefore, the mechanical length of the sliding load, can be decreased for equal accuracy.
- The frequency range can be extended.
- The positioning of the sliding load becomes less critical due to the higher redundancy.

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The authors are with Vrije Universiteit Brussel (VUB/TW-ELEC), B-1050 Brussels, Belgium.

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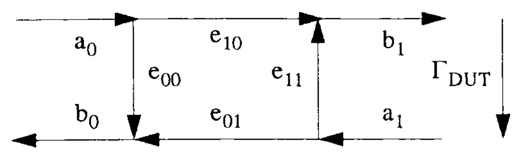

Fig. 1. One-port error-model adaptor.

The term *semiparametric* should be explained prior to setting up the mathematical framework. The standard calibration method does not use any parametric model to relate the measurements at one frequency to the measurements at another frequency. This is why the term *nonparametric* will be used for the standard calibration method. The proposed method uses a parametric model for the delay of the sliding load, i.e., a model expresses the interrelation of the phases introduced by the delay as a function of the considered frequency and as a function of the position of the sliding load. It also uses a nonparametric representation for the absorber in the sliding load. Combining these two into one model leads to the term *semiparametric* model.

To set up a general framework, consider the one-port error-adaptor model (Fig. 1). The measured reflection coefficient $\Gamma_0 = b_0/a_0$ equals

$$\Gamma_0 = e_{00} + \frac{e_{01}e_{10}\Gamma_{\mathrm{DUT}}}{1 - e_{11}\Gamma_{\mathrm{DUT}}} \tag{1}$$

with $\Gamma_{\mathrm{DUT}} = a_1/b_1$ being the exact reflection coefficient. The sliding load uses a transmission line with variable length, terminated with a good (but imperfect) load ($\Gamma_{\mathrm{DUT}} = \Gamma_L e^{-j2\tau\omega}$ with $|\Gamma_L| \ll 1$). The error coefficient $|e_{11}|$ is small, with respect to 1, by construction of the vector network analyzer. Hence, Γ_0 can be approximated by $e_{00} + \Gamma e^{-j2\tau\omega}$ with $\Gamma = e_{01}e_{10}\Gamma_L$, i.e., e_{00} equals the center of the circle while $|\Gamma|$ determines the radius of a circle in the complex plane (Fig. 2).

To compute the center of the circle e_{00}, the standard calibration method used in microwave-network analyzers requires at least three distinct points at every frequency. The accuracy of e_{00} decreases drastically if the different points on the

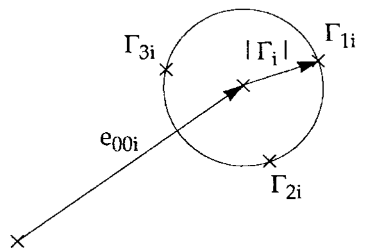

Fig. 2. The circle-fitting problem.

circle form one or two clusters of points. The proposed method exploits the knowledge of the physical model of the transmission line (e.g., for an airline or waveguide [2]). This information is used to interrelate the phase differences over the different frequencies

$$\Gamma_{0ik} = e_{00i} + \Gamma_i e^{j\phi(d_k,\, \omega_i,\, p)} \tag{2}$$

where $i = 1, \cdots, F$ is the index over the frequencies and $k = 1, \cdots, K$ is the index over the different (unknown) positions of the load d_k. The (known) transmission-line parameters are presented by p. This model includes transmission lines with frequency-dependent propagation velocities (e.g., waveguides) and/or known attenuation. The phase model $\phi(d_k,\, \omega_i,\, p)$ makes it possible to use only two distinct points to determine the circle if the different positions d_k are identifiable.

Example

A sliding load built using an coaxial airline has a linear phase response

$$\phi_{\text{coax}}(d_k,\, \omega_i,\, p) = \frac{2\pi d_k}{\lambda} \tag{3}$$

with $\lambda_g = c_0/f_i$ being the wavelength in free space and c_0 being the speed of light. Waveguide transmission lines exhibit a dispersive phase response

$$\phi_{\text{waveguide}}(d_k,\, \omega_i,\, p) = \frac{2\pi d_k}{\lambda_g} \tag{4}$$

where $\lambda_g = \lambda/\sqrt{1 - (\lambda/\lambda_{\text{co}})^2}$ represents the wavelength in the waveguide with λ_{co} representing the cutoff frequency of the waveguide.

This paper is organized as follows. Section II proposes an efficient semiparametric circle-fitting algorithm for the semiparametric model. Section III describes a stochastically based model validation technique. Section IV gives some comments on the minimization algorithm used. Optimal positions on the sliding load when using only three positions are determined in Section V. Finally, Section VI experimentally verifies the proposed method.

II. Modeling

A. Semiparametric

To obtain an efficient semiparametric estimator, a weighted least-squares estimator is introduced, which weights the least-squares error between the measurements $\Gamma_{0ik}^m = \Gamma_{0ik} + n_{ik}$ and the model Γ_{0ik} inverse proportionally with their standard deviation. It assumes that the additive noise is zero mean, i.e., $E[n_{ik}] = 0$ with $E[x]$ being the expected value of x.

The sample variances $\hat{\sigma}_{ik}^2$ obtained from repeated independent experiments (keeping the setting of the position constant) serve as estimates of the true noise variances σ_{ik}^2. The sample mean of independent experiments are used as the measurements of Γ_{0ik}^m [3]. Hence, the proposed weighted least-squares cost becomes

$$L_K(e_{00},\, \Gamma_i,\, d_k) = \sum_{i=1}^{F} \sum_{k=1}^{K} \frac{|\Gamma_{0ik}^m - [e_{00i} + \Gamma_i e^{j\phi(d_k,\, \omega_i,\, p)}]|^2}{\hat{\sigma}_{ik}^2}. \tag{5}$$

Minimizing (5) with respect to the parameters $d_k,\, \Gamma_i$, and e_{00i} provides the semiparametric estimates. One position should be chosen in order to regularize the minimization problem (e.g., $d_1 = 0$). The large number of unknown parameters ($4F + K - 1$ real parameters) in this least-squares optimization problem is the main reason for looking for numerical methods with reduced complexity. This can be done by eliminating the parameters e_{00i} and Γ_i, since the error vector is linear in these parameters [4]–[6]. This reduces the complexity of the optimization problem of $4F + K - 1$ down to $K - 1$ real parameters—namely the different positions. Therefore, (5) is rewritten in the following matrix notation:

$$\sum_{i=1}^{F} (\mathbf{X}_i - \mathbf{A}_i\mathbf{P}_i)^h \mathbf{C}_i (\mathbf{X}_i - \mathbf{A}_i\mathbf{P}_i) \tag{6}$$

with

$$\mathbf{A}_i = \begin{bmatrix} 1 & e^{j\phi(d_1,\, \omega_i,\, p)} \\ \cdots & \cdots \\ 1 & e^{j\phi(d_K,\, \omega_i,\, p)} \end{bmatrix} \in \mathbb{C}^{K \cdot 2} \tag{7}$$

and $\mathbf{X}_i^t = [\Gamma_{0i1}^m, \cdots, \Gamma_{0iK}^m] \in \mathbb{C}^{1 \cdot K}$, $\mathbf{C}_i = \text{diag}\,(\hat{\sigma}_{i1}^{-2}, \cdots, \hat{\sigma}_{iK}^{-2}) \in \mathbb{R}^{K \times K}$, $\mathbf{P}_i^t = [e_{00i}, \Gamma_i] \in \mathbb{C}^{1 \cdot 2}$, $\cdot^t$ the matrix transpose, $\cdot^h$ the Hermitian matrix transpose, $\cdot^{-1}$ the matrix inverse of a square matrix and where diag$(\cdot)$ returns a diagonal matrix with the argument on the diagonal [7]. Minimizing (6) with regard to $\mathbf{P}_i$ gives

$$\mathbf{P}_i = (\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i)^{-1} \mathbf{A}_i^h \mathbf{C}_i \mathbf{X}_i. \tag{8}$$

Elimination of $\mathbf{P}_i$ in (6) gives

$$Cte + \sum_{i=1}^{F} \mathbf{X}_i^h \mathbf{C}_i \mathbf{A}_i (\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i)^{-1} \mathbf{A}_i^h \mathbf{C}_i \mathbf{X}_i \tag{9}$$

where Cte is independent of d_k. Hence

$$\hat{d}_k = \arg\min_{d_k} \sum_{i=1}^{F} \mathbf{X}_i^h \mathbf{C}_i \mathbf{A}_i (\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i)^{-1} \mathbf{A}_i^h \mathbf{C}_i \mathbf{X}_i. \tag{10}$$

The estimated parameters $\hat{\mathbf{P}}_i$ are given by evaluating (8) in the estimated $\hat{d}_k$. The stochastic properties of $\hat{\mathbf{P}}_i$ are determined by $\mathbf{X}_i$ and indirectly (through $\mathbf{A}_i$) by the stochastics of the estimates $\hat{d}_k$. The variance on $\hat{d}_k$ decreases to zero as $1/F$

when the number of frequencies tends to infinity. Hence, the covariance matrix of $\hat{\mathbf{P}}_i$ tends toward

$$(\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i)^{-1} \tag{11}$$

for $F \rightarrow \infty$. Note that the measured noise spectra $\hat{\sigma}_{ik}^2$ may vary with the frequency.

To study the efficiency of the semiparametric method, assume that the noise variances $\hat{\sigma}_{ik}^2$ are independent of the positions d_k (i.e., $\forall k$: $\hat{\sigma}_{ik}^2 = \sigma_i^2$). The determinant of $\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i$ is then given by

$$\det(\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i) = \sigma_i^{-4} \left\{ K^2 - \sum_{k1=1}^{K} \sum_{k2=1}^{K} \cos[\phi(d_{k1}, \omega_i, p) - \phi(d_{k2}, \omega_i, p)] \right\}. \tag{12}$$

Hence, the complex variance on the estimates of the e_{00i}, given by (11), can therefore, be approximated by $K\sigma_i^{-2} / \det(\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i)$.

B. Nonparametric

The standard nonparametric estimation algorithm is given in [1] and minimizes $\sum_{k=1}^{K} (|\Gamma_{0ik}^m - e_{00i}|^2 - \rho_i^2)^2$ or equivalently

$$\sum_{k=1}^{K} [|\Gamma_{0ik}^m|^2 - 2\,\mathrm{Re}(\Gamma_{0ik}^m)\,\mathrm{Re}(e_{00i}) - 2\,\mathrm{Im}(\Gamma_{0ik}^m)\,\mathrm{Im}(e_{00i}) - (\rho_i^2 - |e_{00i}|^2)]^2. \tag{13}$$

Here, ρ represents the radius of the circle in the complex plane. The nonlinear least-squares problem can be transformed into a linear least-squares problem through the parametrization $\mathbf{P}_i^t = [\mathrm{Re}(e_{00i}),\ \mathrm{Im}(e_{00i}),\ \rho_i^2 - |e_{00i}|^2]$, $\mathbf{X}_i^t = [|\Gamma_{0i1}^m|^2, \cdots, |\Gamma_{0iK}^m|^2]$, $(\mathbf{X}_i - \mathbf{A}_i\mathbf{P}_i)^t(\mathbf{X}_i - \mathbf{A}_i\mathbf{P}_i)$ with

$$\mathbf{A}_i = \begin{bmatrix} 2\,\mathrm{Re}(\Gamma_{0i1}^m) & 2\,\mathrm{Im}(\Gamma_{0i1}^m) & 1 \\ \cdots & & \cdots \\ 2\,\mathrm{Re}(\Gamma_{0iK}^m) & \mathrm{Im}(\Gamma_{0iK}^m) & 1 \end{bmatrix}. \tag{14}$$

Hence, the estimates are given by $(\mathbf{A}_i^t \mathbf{A}_i)^{-1} \mathbf{A}_i^t \mathbf{X}_i$. A detailed sensitivity study of the estimates can be found in [1]. This reference also clearly explains why it is favorable to use more than four positions when performing a broad-band calibration using a sliding load.

C. Comparing the Uncertainty on the Estimates

Consider the case where the K positions d_k are equidistantly distributed along the circle, and assume for simplicity that all noise sources satisfy $\sigma_{ik}^2 = \sigma^2$ for all ik. Reference [1] proves that the complex variance on the estimated e_{00i} equals $2\sigma^2/K$ for the nonparametric case. The variance on the estimates using the semiparametric method approaches σ^2/K as $F \rightarrow \infty$. Hence, in this particular case, the number of K positions for the semiparametric method can be reduced with a factor 2 without loss in accuracy with respect to the nonparametric case.

III. Detection of Defects on the Sliding Load

A sliding load for calibration purposes is hard to design from a mechanical point of view. Hence, detection of modeling errors is no luxury. The easiest way to validate the model with the measurements is by comparing the residual errors with the noise level. These residual errors equal the measured values minus the estimated model

$$\Gamma_{0ik}^m - [e_{00i} + \Gamma_i e^{j\phi(d_k, \omega_i, p)}]. \tag{15}$$

If the model is valid, then the residual errors should be comparable with the noise level. This means for the mean-square error of the residual errors

$$\mathrm{MSE}_i^S = \frac{1}{K} \sum_{k=1}^{K} |\Gamma_{0ik}^m - [e_{00i} + \Gamma_i e^{j\phi(d_k, \omega_i, p)}]|^2 \tag{16}$$

for the semiparametric model should be compared with the mean-measured-noise variance

$$\overline{\sigma}_i^2 = \frac{1}{K} \sum_{k=1}^{K} |\sigma_{ik}^2|^2 \tag{17}$$

at the ith frequency. Reference [1] gives a closed-form expression for the estimated noise level in the nonparametric case, assuming a large signal-to-noise ratio and for $K \gg 3$

$$\mathrm{MSE}_i^N = \frac{1}{4R_i^2 K} \sum_{k=1}^{K} (|\Gamma_{0ik}^m - e_{00i}|^2 - R_i^2)^2. \tag{18}$$

Comparing MSE_i^S and MSE_i^N with $\overline{\sigma}_i^2$ reveals possible modeling errors. The advantage of the semiparametric model is that model validation is possible for all $K \geq 3$ while the nonparametric method, based on (18), assumes that K is much larger than 3.

IV. Computational Aspects

Equation (10) can be minimized through standard Gauss–Newton-type minimization algorithms [4], [6] (e.g., a Levenberg–Marquardt algorithm). These minimization algorithms are available in both commercial packages (e.g., a MATLAB Toolbox)[1] and public domain packages (e.g., minpack).[2] These minimization methods require that (10) is rewritten as a nonlinear least-squares problem $\mathbf{e}^h\mathbf{e}$, where $\mathbf{e} \in \mathbb{C}^{F \cdot 1}$ denotes the error vector whose ith component is given by

$$\mathbf{e}_{[i]} = \mathbf{H}_i \mathbf{C}_i^{1/2} \mathbf{X}_i \tag{19}$$

with

$$\mathbf{H}_i = \mathbf{C}_i^{1/2} \mathbf{A}_i (\mathbf{A}_i^h \mathbf{C}_i \mathbf{A}_i)^{-1} \mathbf{A}_i^h \mathbf{C}_i^{1/2}. \tag{20}$$

Gauss–Newton-type minimization algorithms are iterative and require the knowledge of the derivatives of the error vector with regard to the parameters d_k. These derivatives are stored

[1] The MathWorks, Inc., *Optimization Toolbox User's Guide*, 1992.

[2] J. J. Moore, B. S. Garbow, and K. E. Hillstrom, *User's Guide for MINPACK-1*, 1980.

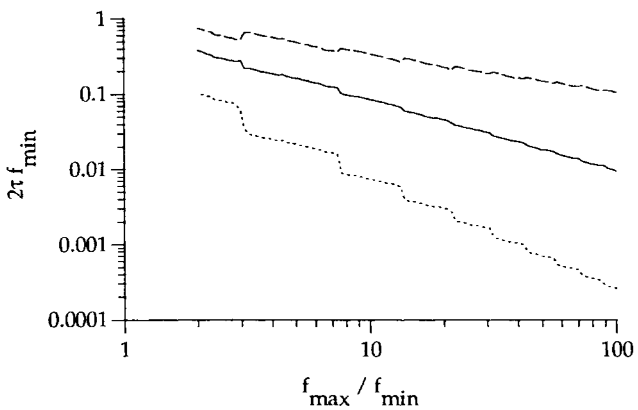

Fig. 3. Optimal delay setting $2\tau_2 f_{\min}$ and $2\tau_3 f_{\min}$ (in dashed and full line, respectively), and the 3-dB uncertainty bound $2\delta\tau f_{\min}$ (dotted line) as a function of the ratio $f_{\max}/f_{\min}$.

into the Jacobian matrix $\mathbf{J} \in \mathbb{C}^{F \cdot K}$. The i, kth element of $\mathbf{J}$ can be written as

$$\mathbf{J}_{[i,k]} = \left[(\mathbf{I}-\mathbf{H}_i)\mathbf{C}_i^{1/2}\frac{\partial \mathbf{A}_i}{\partial d_k}\mathbf{G}_i + \mathbf{G}_i^h \frac{\partial \mathbf{A}_i^h}{\partial d_k}\mathbf{C}_i^{1/2}(\mathbf{I}-\mathbf{H}_i)\right]\mathbf{C}_i^{1/2}\mathbf{X}_i \quad (21)$$

where $\mathbf{G}_i = (\mathbf{A}_i^h\mathbf{C}_i\mathbf{A}_i)^{-1}\mathbf{A}_i^h\mathbf{C}_i^{1/2}$ and $\mathbf{I}$ is the identity matrix. From a numerical point of view, it is an extremely bad idea to compute $\mathbf{G}_i$ and $\mathbf{H}_i$ through the explicit formulas given above. The main argumentation is that the straightforward computation of $(\mathbf{A}_i^h\mathbf{C}_i\mathbf{A}_i)^{-1}\mathbf{A}_i^h\mathbf{C}_i^{1/2}$ is more sensible to rounding errors than when using, for example, a $\mathbf{QR}$ factorization [6], [7]. The $\mathbf{QR}$ factorization of $\mathbf{C}_i^{1/2}\mathbf{A}_i$ computes the matrices $\mathbf{Q}_i$ and $\mathbf{R}_i$ in a numerically stable way such that $\mathbf{Q}_i\mathbf{R}_i = \mathbf{C}_i^{1/2}\mathbf{A}_i$ with $\mathbf{Q}_i^h\mathbf{Q}_i = \mathbf{I}$ and $\mathbf{R}_i$ being an upper triangular. Hence, $\mathbf{G}_i$ and $\mathbf{H}_i$ are computed in a numerically stable way using $\mathbf{G}_i = \mathbf{R}_i^{-1}\mathbf{Q}_i^h$ and $\mathbf{H}_i = \mathbf{Q}_i\mathbf{Q}_i^h$ [7]. After the minimization of the cost with respect to the positions d_k, the parameters $\mathbf{P}_i$ can be computed using $\mathbf{P}_i = \mathbf{G}_i\mathbf{C}_i^{1/2}\mathbf{X}_i$.

V. Optimal Positions

The main goal of this section is to propose three optimal positions. This means that only three positions on the sliding load are required to compute all e_{00i} at all frequencies in a given frequency range. Therefore, the following topics are considered.

- The transmission line is a pure delay $\phi_{ik} = -4\pi\tau_k f_i$.
- Only three positions are required for a given frequency band $[f_{\min}, f_{\max}]$. Hence, only two delays, $\tau_2 < \tau_3$ ($\tau_1 = 0$) need to be considered.
- The problem must remain regular for all frequencies $f_{\min} \leq f_i \leq f_{\max}$.
- The maximal variance on the estimates of e_{00i} must be minimal.
- The required accuracy for the positioning must be achievable in practice.

To satisfy the third requirement, all three points on the circle should never coincide for any frequency $f_i \in [f_{\min}, f_{\max}]$. To ensure at least two distinct points, the minimal delay, τ_2, should be chosen such that for every $f_i \in [f_{\min}, f_{\max}]$, $2\tau_2 f_i \notin \mathbb{N}$ irrespective of τ_3. A sufficient condition is that $2\tau_2 f_{\max} < 1$.

The set of optimal delays is determined in a minimax sense on the variance of the estimates of e_{00i}. This is equivalent with maximizing (over the positions) the minimum (over the frequencies) of the determinant given by (12). The set of optimal delays as a function of the ratio $f_{\max}/f_{\min}$ are given in Fig. 3. These are obtained using a classical simplex optimization method. The uncertainty-bound $\delta\tau$ corresponds with the maximum change on τ_2 and τ_3, resulting in an increase of the variance of, at most, 3 dB. This illustrates the robustness with respect to positioning errors.

VI. Experimental Results

The semiparametric method is verified on measurements obtained using an HP-8510C network analyzer. The reflection coefficient of the sliding load of the HP-85 056A calibration kit is measured at 201 frequencies from 45-to-50-GHz in stepped mode with the number of averages equal to 128. For every position, ten individual measurements are performed to obtain the stochastic properties of the measurement noise. This is repeated for 26 equispaced positions from 0 to 25 mm.

A. The Reference Parameters

Exact parameter values are required to compare the efficiency of the estimates of the nonparametric and the semiparametric methods. Since both methods are compared on measured data, no exact values are available. Therefore, the estimated e_{00} using *all* 26 positions are used as the reference values. The estimates are computed using the 185 measurements from 4 to 50 GHz. The measurements below 4 GHz are rejected since the sliding load used is not specified below 4 GHz.

The following notations will be used: superscript N (for nonparametric) or S (for semiparametric) followed by the number K. Hence, e_{00}^{S26} stands for the reference parameters using the semiparametric method. The worst-case accuracy of a given method is defined as the worst-case difference between the obtained e_{00}^x and e_{00}^{S26}, i.e.,

$$\max_i \left(|e_{00i}^x - e_{00i}^{S26}|\right) \quad (22)$$

with $x \in \{N6, N26, S3, S6\}$. Fig. 4 shows a typical measurement of the reflection factor Γ_{0i}^m together with its noise level σ_i^2. Fig. 5 compares the mean noise level $\overline{\sigma}_i^2$ with the mean-square of the residuals MSE_i^{S26} for e_{00}^{S26}. It shows that almost no modeling errors are visible. Although the sliding load is specified in the frequency band from 4 to 50 GHz, there are small modeling errors present just above 4 GHz.

B. Comparison of the Methods

The nonparametric and semiparametric methods are compared as follows. The nonparametric model uses six out of the 26 positions. The differences in position Δd_k are set equal to the ones indicated on the sliding load. The semiparametric model first uses the optimal positions, i.e., differences Δd_k of

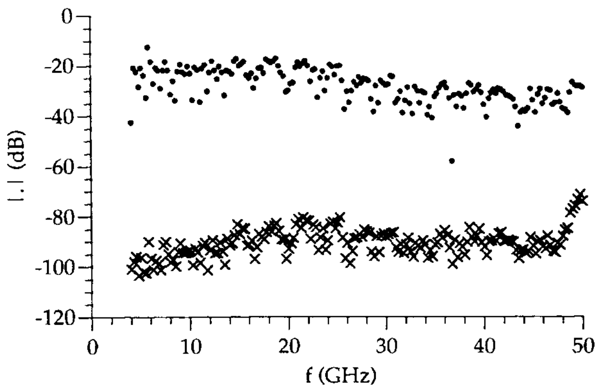

Fig. 4. The measured raw data (dots) and its standard deviation, both expressed in decibels as a function of the frequency.

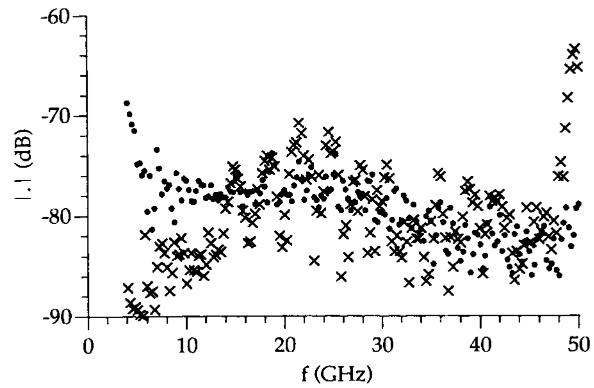

Fig. 5. The mean-square residuals MSE_i^{S26} of the reference model S_{00}^{S26} (dots) and the mean noise level $\overline{\sigma}_i^2$, both expressed in decibels as a function of the frequency.

2.66 and 10.64 mm. Afterwards, the semiparametric model is identified using the same six positions as used for the nonparametric model. To compare the efficiency of the different methods, the mean-square error of the estimates e_{00}^{S3}, e_{00}^{S6}, e_{00}^{N6}, and e_{00}^{N26} are considered as follows:

$$\frac{1}{F}\sum_{i=1}^{F}|e_{00}^{x}-e_{00}^{S26}|^2 \tag{23}$$

with $e_{00}^{x} \in \{e_{00}^{S3}, e_{00}^{S6}, e_{00}^{N6}, e_{00}^{N26}\}$. Table I clearly shows the following.

- There is a good correspondence between the semiparametric and the nonparametric method using all 26 positions.
- The semiparametric method using three positions e_{00}^{S3} and the nonparametric method using six positions e_{00}^{N6} have approximately the same accuracy.
- For the same number of positions, the mean-square error for the semiparametric model is half the mean-square error for the nonparametric model (i.e., 3 dB).

C. Robustness of the Positioning

Fig. 6 demonstrates the robustness of the semiparametric method with respect to the positioning. The black part of the upper plot represents the region in the τ_2–τ_3 plane where

TABLE I
THE MEAN-SQUARE ERROR OF THE ESTIMATES [SEE (23)] EXPRESSED IN DECIBELS

e_{00}^{S3}	e_{00}^{S6}	e_{00}^{N6}	e_{00}^{N26}
-79.5	-81.5	-78.2	-86.4

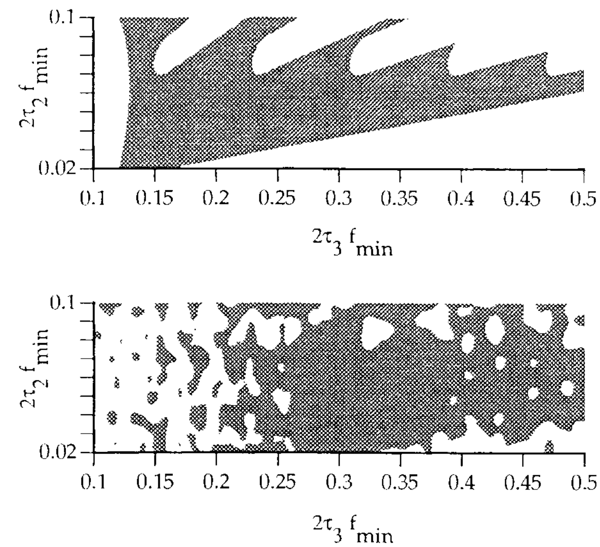

Fig. 6. Optimal positioning. The upper plot represents the theoretical variance. The lower plot represents the worst-case accuracy of the semiparametric method using three positions compared with the worst-case accuracy of the nonparametric method using six positions.

(for $f_{\max} = 12.5 f_{\min}$) the theoretical variance is, at most, 6 dB above the optimal setting, i.e., $2\tau_2 f_{\min} = 0.0709$ and $2\tau_3 f_{\min} = 0.2836$. The lower plot represents the region where the worst-case accuracy of the semiparametric method using three positions $\max_i(|e_{00i}^{S3} - e_{00}^{S26}|)$ is, at most, 6 dB above the mean worst-case accuracy using the nonparametric method with the six positions marked on the sliding load (i.e., -72 dB $\pm$ 3 dB). Halving the number of positions inevitably increases the uncertainty by 3 dB. This, together with the uncertainty of 3 dB on $\max_i(|e_{00i}^{N6} - e_{00i}^{S26}|)$, explains why a 6-dB boundary has been chosen. The plot clearly illustrates the robustness of the semiparametric method and shows that the results obtained using the three optimal positions are as accurate as when using six positions with the nonparametric method.

D. Detection of Modeling Errors

Both the nonparametric and the semiparametric methods are used to detect modeling errors on the measurements. This is done by comparing the mean noise variance $\overline{\sigma}_i^2$ with (16) and (18) for the measured data in the frequency band of 45 to 50 GHz. The main goal is to determine the frequency band in which the sliding load fulfills its specification. This should be from 4 to 50 GHz as specified by HP.

Fig. 7 represents the mean-square error of the semiparametric model MSE_i^{S6} expressed in decibels for $K = 3$. It also shows the mean-measured-noise variance $\overline{\sigma}_i^2$. This clearly illustrates that the modeling errors below 4 GHz can be detected using the semiparametric method, even when only

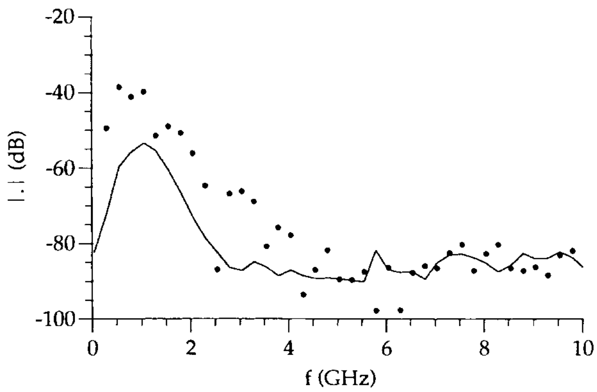

Fig. 7. The mean-square residuals MSE_i^{S3} of the semiparametric model using three positions S_{00}^{S3} (dots) and the mean noise level $\overline{\sigma}_i^2$ (full line), both expressed in decibels as a function of the frequency. Modeling errors below 4 GHz are visible even with three positions.

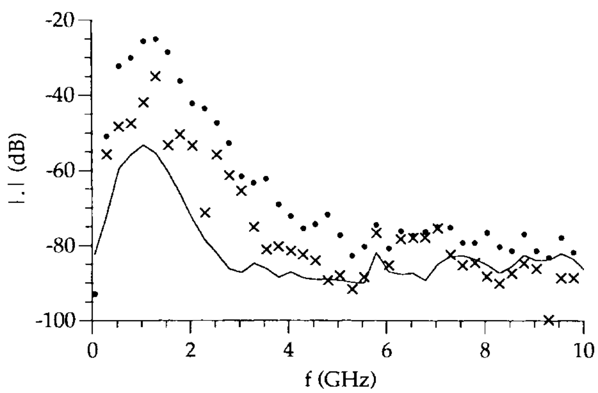

Fig. 8. The mean-square residuals MSE_i^{S6} of the semiparametric model S_{00}^{S6} (dots); the mean-square residuals MSE_i^{N6} of the nonparametric model S_{00}^{N6} (both six positions), and the mean noise level $\overline{\sigma}_i^2$ (full line).

three positions are used. Fig. 8 represents both MSE_i^{S6} and MSE_i^{N6} together with $\overline{\sigma}_i^2$. Both cases use six positions. It can be concluded that the standard nonparametric method can also be used for model validation, although the number of measured positions need to be sufficiently large. Fig. 8 also illustrates that the semiparametric method provides a more sensible validation technique in the sense that defects in the sliding load are detected more easily.

E. Computational Aspects

The only drawback of the proposed method is the computational load. The minimization methods typically require 5 to 10 iterations on the measurements from 4 to 50 GHz to converge to the minimum when the starting values from the positions are given with an accuracy of 1 mm. Every iteration requires—depending on the implementation used—about 10–40 times more floating point operations (flops) than the nonparametric method. Hence, the semiparametric method requires more flops of one or two orders of magnitude than the nonparametric method. Considering the performances of the commercial computers today, the loss in time due to the increased computational complexity is compensated by halving the number of required positions. All computation were done in MATLAB on a PowerPC Macintosh 8100/80 using a nonoptimized code. The estimation of e_{00}^{N6} and e_{00}^{S3} required 1 s (45 Kflops) and 13.6 s (3287 Kflops), respectively.

VII. Conclusion

A new calibration algorithm is introduced to compute the error coefficients e_{00} of a microwave-network analyzer using the measurements on a sliding load. The proposed semiparametric technique takes the phase relationships over the frequencies into account, assuming that the mathematical model of the transmission lines is known. The values of the (variable) positions are estimated together with the parameters of the circle. This produces more accurate results, requires only three positions for the whole frequency band, and is more robust to the choice of the positions. Using the variance of the estimates, optimal positions are proposed. Furthermore, systematic errors in the calibration element can be detected more easily. The performance of the proposed method is illustrated on measurements from 4 to 50 GHz, showing that the accuracy of the semiparametric method using three positions is comparable with the nonparametric method using six positions. Measurements from 45 MHz to 50 GHz demonstrated the ability of detecting defects in the sliding load, even when only three positions on the sliding load are measured.

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Gerd Vandersteen received the degree of electrical engineer (burgerlijk ingenieur) from the Vrije Universiteit Brussel (VUB), Brussels, Belgium, in 1991, and an additional degree in telecommunication, from the Université Catholique de Louvain (UCL), Louvain-la-Neuve, Belgium, in 1992.

He is presently Aspirant of the National Fund for Scientific Research (NFWO) in the Electrical Measurement Department (ELEC) of the VUB. His main interests are parameter estimation/system identification and microwaves.

Yves Rolain (M'90) received the degree of electrical engineer (burgerlijk ingenieur), the degree of computer sciences, and the degree of doctor in applied sciences, all from the Vrije Universiteit Brussel (VUB), Brussels, Belgium, in 1984, 1986, and 1993, respectively.

He is presently in the Electrical Measurement Department (ELEC) of the VUB. His main interests are applied digital-signal processing and parameter estimation/system identification.

Johan Schoukens (M'90–SM'92–F'97) received the degree of engineer, and the degree of doctor in applied science, from the Universiteit Brussel (VUB), Brussels, Belgium in 1980 and 1985, respectively.

He is presently Research Director of the National Fund for Scientific Research (NFWO) and part-time Lecturer at the VUB. His research interests are in the field of identification of linear and nonlinear systems.

Ann Verschueren received the degree of electrical engineer (burgerlijk ingenieur) from the Vrije Universiteit Brussel (VUB), Brussels, Belgium, in 1994.

She is presently working in the Electrical Measurement Department (ELEC) at the VUB. Her main interest is the parametric metallization of high-frequency systems.